Three-Phase Load Flow Methods for Radial Distribution Networks

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Abstract— A distribution system has certain distinguishing features which make it different and somewhat difficult to analyze as compared to a transmission system. Unbalanced loads, untransposed lines, single-phase and two-phase laterals are some of them. The distribution system needs to be analyzed on the three-phase basis instead of the single-phase basis. Hence, the three-phase load flow for the distribution system is significantly different from the conventional load flow for the transmission system. In this paper, various load flow methods for distribution systems are reviewed. These methods are applied on a sample 8-bus system. The results are reported. The performance of these methods is compared for various parameters.

1. INTRODUCTION

A load flow is an essential tool for the steady state analysis of any power system. The main objective of the load flow analysis is to find out the real and reactive powers flowing in each line along with the magnitude and phase angle of the voltage at each bus of the system for the specific loading conditions. The paper focuses on the various load flow methods for the distribution systems. The distribution system has some characteristic features like radial structure, untransposed lines, large number of nodes, and unbalanced loads along with single-phase and two-phase laterals. These features need to be taken into account while carrying out the load flow analysis of the distribution system. The system needs to be analyzed on the three-phase basis unlike the transmission system which can be analyzed on the per-phase basis.

Various methods are available in the literature, which exploit the topology (radial structure) of the distribution system to carry out the load flow analysis. These load flow methods can be divided into two categories. The first category consists of different versions of Newton-Raphson method [1- 4]. The other methods, for the radial distribution systems, like Gauss-Seidel method are classified into the second category [5-9]. The implicit Z-bus method [5] works on the principle of superposition. A modified Gauss-Seidel method is the blend of the implicit Z-bus method and the Gauss-Seidel method [6]. A network topology based method uses two matrices, viz. bus-injection to branch-current (BIBC) and branch-current to bus-voltage (BCBV) matrices, to find out the solution [7]. The forward-backward substitution [8] and ladder network theory [9] based approaches trace the network to and fro from its load end to source end.

This paper reviews the algorithms of various methods of three-phase load flow. The methods are: A) Implicit Z-bus method B) Modified Gauss-Seidel method C) Network topology based method D) Forward-backward substitution method E) Ladder network theory. The paper presents the results of the network topology based method, forward-backward substitution and ladder network theory. These three methods are compared on the basis of the obtained results.

2. MODELING OF DISTRIBUTION SYSTEM COMPONENTS

The individual components of a distribution system are modeled by their mathematical equivalents. The three-phase modeling of distribution system components is given in [9]. The series impedance matrix of a three-phase line section is given by equation 1.

$$Z_{abc} = \begin{bmatrix} z_{aa-n} & z_{ab-n} & z_{ac-n} \\ z_{ba-n} & z_{bb-n} & z_{bc-n} \\ z_{ca-n} & z_{cb-n} & z_{cc-n} \end{bmatrix} \tag{1}$$

This equation is obtained after Kron's reduction. It takes care of the effects of the neutral or ground. At each bus i, the complex power S_i is given by,

$$S_i = P_i^{spec} + j\, Q_i^{spec} \tag{2}$$

where P_i^{spec} and Q_i^{spec} are the specified real and reactive powers respectively of bus i. The equivalent current injection at bus i for the k^{th} iteration is given as,

$$I_i^k = \left(\frac{P_i^{spec} + jQ_i^{spec}}{V_i^{(k)}} \right)^* \tag{3}$$

3. THREE-PHASE DISTRIBUTION LOAD FLOW ANALYSIS

Most of the distribution systems are radial in nature with a single voltage source. This special property of the distribution system is used to derive various formulations. Different iterative methods similar to Gauss-Seidel method are discussed in this paper. In this section, the algorithms for these methods are given.

A. Implicit Z-bus Method

The implicit Z-bus method is the most commonly used method [5]. The method works on the principle of superposition as applied to the system bus voltages. According to the principle of superposition, only one type of source is considered at a time for the calculation of bus voltages. An iterative procedure is used in this method. Initially, all the bus voltages are assumed to be equal to the swing bus voltage (only swing bus is considered as the source in the system with all the current injections at load buses taken as zero). In the next step, since the current injections and bus voltages are dependent on each other, these quantities are required to be determined iteratively. The swing bus is short-circuited while calculating the component of bus voltages due to the current injections.

The following steps are involved in this algorithm:

1. The bus voltages are assumed to have some initial value. The Y-bus (Y_B) is formed.
2. The current injections are computed by using equation 3 for which the recent values of bus voltages are taken.
3. The voltage deviations (VD) due to current injections are computed by the factorization of Y-bus,

$$I^k = [Y_B]\,[VD]^k \tag{4}$$

4. The voltage deviations calculated in step 3 are superimposed on the no load bus voltage (V_{NL}). Hence, the bus voltages are updated as,

$$V^{k+1} = V_{NL} + [VD]^k \tag{5}$$

5. The convergence is checked. If the method has not converged, then steps from 2 to 4 are repeated.

B. Modified Gauss-Seidel Method

The implicit Z-bus method described earlier requires the factorization of the full Y-bus matrix, adversely affecting the performance in terms of speed. Hence, a new method has been suggested in [6] by blending the implicit Z-bus method and the Gauss-Seidel method to improve the computational efficiency.

For a distribution system with n buses, where P_i^{spec} and Q_i^{spec} are the specified powers at bus i, the bus voltage for k^{th} iteration can be calculated by using the Gauss-Seidel method as,

$$V_i^{(k)} = \frac{1}{Y_{ii}}\left[\frac{P_i^{spec} - jQ_i^{spec}}{V_i^{(k-1)*}} - \sum_{j=1}^{i-1} Y_{ij}V_j^k - \sum_{j=i+1}^{n} Y_{ij}V_j^{(k-1)}\right] \tag{6}$$

The values of voltages on the right hand side of equation 6 are the most recently computed values. The same equation for i^{th} bus can be written in the matrix form as,

$$\begin{bmatrix} Y_{11} & -- & Y_{1i} & -- & Y_{1n} \\ -- & -- & -- & -- & -- \\ Y_{i1} & -- & Y_{ii} & -- & Y_{in} \\ -- & -- & -- & -- & -- \\ Y_{n1} & -- & Y_{ni} & -- & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1^k \\ -- \\ V_i^{k-1} \\ -- \\ V_n^{k-1} \end{bmatrix} = \begin{bmatrix} I_1 \\ -- \\ I_i \\ -- \\ I_n \end{bmatrix} \tag{7}$$

where each element of Y-bus is a 3×3 matrix. For example,

$$Y_{11} = \begin{bmatrix} Y_{11aa} & Y_{11ab} & Y_{11ac} \\ Y_{11ba} & Y_{11bb} & Y_{11bc} \\ Y_{11ca} & Y_{11cb} & Y_{11cc} \end{bmatrix}$$

Accordingly, the Y-bus in equation 7 can be rearranged as,

$$Y_B = \begin{bmatrix} Y_{AA} & Y_{AB} & Y_{AC} \\ Y_{BA} & Y_{BB} & Y_{BC} \\ Y_{CA} & Y_{CB} & Y_{CC} \end{bmatrix} \tag{8}$$

where each element is n×n matrix.

Equation 4 can be written as,

$$\begin{bmatrix} Y_{AA} & Y_{AB} & Y_{AC} \\ Y_{BA} & Y_{BB} & Y_{BC} \\ Y_{CA} & Y_{CB} & Y_{CC} \end{bmatrix} \begin{bmatrix} VD_A \\ VD_B \\ VD_C \end{bmatrix} = \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{9}$$

where A, B and C are the three phases of the distribution system.

Using equation 7 in equation 9 we get,

$$Y_{AA} * VD_A^k + Y_{AB} * VD_B^{k-1} + Y_{AC} * VD_C^{k-1} = I_A \tag{10a}$$

$$Y_{BA} * VD_A^k + Y_{BB} * VD_B^k + Y_{BC} * VD_C^{k-1} = I_B \tag{10b}$$

$$Y_{CA} * VD_A^k + Y_{CB} * VD_B^k + Y_{CC} * VD_C^k = I_C \tag{10c}$$

Equation (10a) can be rewritten as,

$$VD_A^k = Y_{AA}^{-1}(I_A - Y_{AB} * V_B^{k-1} - Y_{AC} * V_C^{k-1})$$

Applying the principle of superposition, the bus voltage can be found as,

$$V_A^k = V_{A,NL} + VD_A{}^k \tag{11a}$$

Similarly,

$$VD_B^k = Y_{BB}^{-1}(I_B - Y_{BA} * V_A^k - Y_{BC} * V_C^{k-1})$$

$$V_B^k = V_{B,NL} + VD_B{}^k \tag{11b}$$

$$VD_C^k = Y_{CC}^{-1}(I_C - Y_{CA} * V_A^k - Y_{CB} * V_B^k)$$

$$V_C^k = V_{C,NL} + VD_C{}^k \tag{11c}$$

The values of voltages used in the modified Gauss-Seidel method are the most recently computed values, whereas the

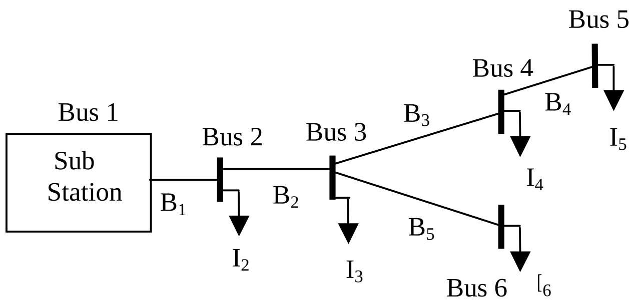

Fig. 1 Sample Distribution System

values of voltages used in the implicit Z-bus method are the values obtained in the previous iteration.

C. Network Topology Based Method

The two methods described earlier use the complete or fractional factorization of Y-bus matrix. Another method, proposed in [7], does not require such factorization. The topographical approach has been used in this method to tackle the problem of load flow. Two matrices are developed, viz. the bus injection to branch current (BIBC) matrix and branch current to bus voltage (BCBV) matrix. By using simple matrix multiplication of these two matrices, the load flow solution is obtained.

Two developed matrices, BIBC and BCBV are used to obtain the load flow solution. The development of these two matrices is explained with reference to Fig. 1. The figure shows a simple distribution system. It has sub-station at its bus number 1, and bus numbers 2 to 6 are the load buses.

A set of equations can be written by applying KCL to the network. The branch currents can be expressed as a function of equivalent current injections. The currents in B_1 to B_5 can be expressed as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

The same equation can be expressed as,

$$[B] = [BIBC]\,[I] \tag{12}$$

The relations between the branch currents and the bus voltages can be obtained by using the following equations:

$$V_2 = V_1 - B_1 Z_{12} \tag{13}$$

$$V_3 = V_2 - B_2 Z_{23} \tag{14}$$

$$V_4 = V_3 - B_3 Z_{34} \tag{15}$$

where Z_{12}, Z_{23} and Z_{34} are the phase impedance matrices of line sections 1-2, 2-3 and 3-4 respectively. By substituting equation 13 and 14 into equation 15, the voltage of bus 4 can be rewritten as,

$$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34} \tag{16}$$

Thus, the bus voltage can be expressed as a function of the branch currents, line parameters and swing bus voltage. By adopting the similar procedure for other buses, the BCBV matrix for the network of Fig. 1 can be derived as,

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} \tag{17}$$

The above equation can be written as,

$$[\Delta V] = [BCBV]\,[B] \tag{18}$$

After the development of matrices, the relations between the bus current injections and bus voltages can be expressed as,

$$[\Delta V] = [BCBV]\,[BIBC]\,[I]$$

$$[\Delta V] = [DLF]\,[I] \tag{19}$$

where $[DLF] = [BCBV]\,[BIBC]$

$$[\Delta V^{k+1}] = [DLF]\,[I^k] \tag{20}$$

$$V^{k+1} = V_1 - \Delta V^{k+1} \tag{21}$$

where V_1 is the swing bus voltage. From equation 3, we can get the equivalent current injections at the k^{th} iteration and hence the voltage deviation can be obtained by solving equation 3 and 20 iteratively. The voltages are updated at each iteration by using equation 21.

D. Forward-Backward Substitution

In all the previous methods, the voltages at all the buses in the system are calculated in one step, by using the matrices. In forward-backward substitution, the KCL and KVL are applied at each node and branch respectively. By solving these equations iteratively, the solution is obtained [8].

The following steps involved in this method:

Optimal ordering of nodes: Nodes are renumbered according to source node - load node relationship to facilitate the forward and backward substitution. Thus, a forward path is created from the source node to the load node and a backward path is traced from the load node to the source node. The branch node nearer to the source is called as the parent node and the other node is called as the child node. Initially, the flat voltage start is assumed.

Backward substitution: This is used to calculate the current in each branch. The current in the last branch is equal to the current injection at the corresponding end node. The voltage values are kept constant. The network is traced in the backward direction. The currents in all the other branches can be found out by using KCL as given by the equation,

$$\begin{bmatrix} I_a(m) \\ I_b(m) \\ I_c(m) \end{bmatrix} = -\begin{bmatrix} i_{La}(i) \\ i_{Lb}(i) \\ i_{Lc}(i) \end{bmatrix} + \sum_{p \in M} \begin{bmatrix} I_{ap} \\ I_{bp} \\ I_{cp} \end{bmatrix} \qquad (22)$$

where I_a (m), I_b (m) and I_c (m) are the branch currents of line section m, and $i_{La,}$ i_{Lb} and i_{Lc} are the equivalent current injections at the child node (i) of branch m. M is the set of line sections connected to m^{th} branch at its child node (p is the number of a line section which is an element of M).

Forward substitution: This is used to calculate the voltage at each node (starting from the child node of the first branch) by using KVL. The swing bus voltage is set to its specified value. The current in each branch is held constant at the value obtained in the backward substitution. Thus, using the branch currents calculated in the backward substitution, the values of voltages are calculated by using the equation,

$$\begin{bmatrix} V_a(i) \\ V_b(i) \\ V_c(i) \end{bmatrix} = \begin{bmatrix} V_a(j) \\ V_b(j) \\ V_c(j) \end{bmatrix} - \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a(m) \\ I_b(m) \\ I_c(m) \end{bmatrix} \qquad (23)$$

where j and i are the parent node and child node respectively. These values of the voltages are used for calculating the currents by backward substitution in the next iteration.

Check for convergence: The forward and backward substitutions are performed in each iteration of the load flow. The voltage magnitudes at each bus in an iteration are compared with their values in the previous iteration. If the error is within the tolerance limit, the procedure is stopped. Otherwise, the steps of backward substitution, forward substitution and check for convergence are repeated.

E. Ladder Network Theory

The ladder network theory given in [9] is very much similar to the forward-backward substitution method. Though the basic principle of both the methods is same, there are differences in the steps of implementation. In the ladder network theory, the optimal ordering of nodes is done first. In the backward substitution, the node voltages are assumed to be equal to some initial value in the first iteration. The currents in each branch are computed by KCL using equation 22. In addition to the branch currents, the node voltages are also computed by using equation 23. Thus, the value of the swing bus voltage is also determined. This calculated value of the swing bus voltage is compared with its specified value. If the error is within the limit, then the load flow converges; otherwise the forward substitution is performed as explained in the case of forward-backward substitution method. Thus, in the ladder network theory, the bus voltages are calculated twice in the same iteration as compared to only once for the forward-backward substitution method. The convergence is checked in the ladder network theory by comparison between the specified and calculated voltage values of the swing bus, whereas the difference between the values of bus voltages at the present and previous iterations is considered for convergence in the forward-backward substitution method.

4. SAMPLE SYSTEM AND RESULTS

A sample system of 8 buses shown in Fig. 2 has been taken from the Taiwan Power Corporation [6]. The three methods C, D and E described in section 3 are tested on the sample system. The base values of the system are 14.4 kV and 100 kVA. The convergence tolerance specified is 0.001 p.u. The converged solutions (voltage magnitudes) are given in Table 1. The angles are not given for brevity. The number of iterations required for all the three methods are found to be same. However, for a larger system, the number of iterations required for the three different methods may vary.

The methods are now compared on the basis of their performance parameters:

Accuracy: The results obtained by all the three methods are compared with those given in [6] which are obtained using the modified Gauss-Seidel method. For the ladder network theory and forward-backward substitution methods, the maximum deviation from the published results in [6] is 0.0059 p.u. Thus, all the three discussed methods are quite accurate.

Rate of convergence: For all the three methods the load flow converged in 3 iterations for the tolerance of 0.001 p.u. The modified Gauss Seidel method has also converged in three iterations for the same tolerance limit [6]. When the tolerance limit is set as 0.0001, the number of iterations required for the convergence is the same for the three methods but it has increased to 4 in the case of modified Gauss-Seidel method.

Execution time: The execution time is 3.1250 seconds for the network topology based method. It is 2.9220 seconds for the forward-backward substitution method and 2.7340 seconds for the ladder network theory on P-IV computer with 1.6 GHz frequency and 128 MB RAM.

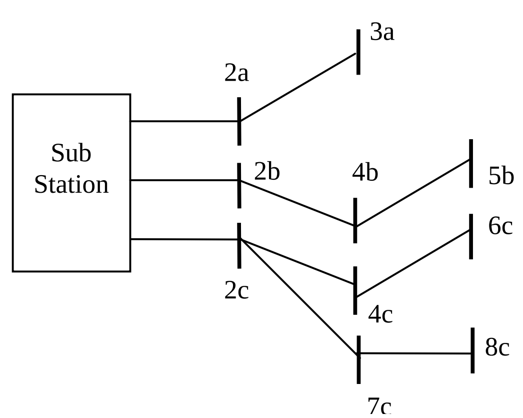

Fig. 2 Sample Distribution system [6]

Table 1 Converged voltage magnitudes (in p.u.)

Bus No.	Phase	Ladder Network Method	Forward Backward Substitution Method	Network Topology based Method
1	a	1.0000	1.0000	1.0000
1	b	1.0001	1.0000	1.0000
1	c	1.0000	1.0000	1.0000
2	a	0.9830	0.9830	0.9839
2	b	0.9714	0.9714	0.9711
2	c	0.9745	0.9745	0.9697
3	a	0.9822	0.9822	0.9832
4	b	0.9655	0.9655	0.9652
4	c	0.9717	0.9716	0.9668
5	b	0.9644	0.9643	0.9640
6	c	0.9697	0.9697	0.9649
7	c	0.9739	0.9739	0.9683
8	c	0.9726	0.9726	0.9671

Flexibility: The degree of flexibility of the methods depends on the ease with which any modification in the system data can be accommodated in their software program. Since the ladder network theory and forward-backward substitution methods work by applying KCL and KVL at each node, any modification can be easily accommodated in them by making change in the corresponding equations. Hence, these methods can be said to be highly flexible in nature. The implicit Z-bus method, modified Gauss-Seidel method and network topology based method use the matrices for their operation. Therefore, if there is any change in the system data, then these matrices are required to be restructured, and hence the flexibility of these methods is lower.

5. CONCLUSION

The load flow for the distribution systems is more involved than the transmission systems due to untransposed lines, unbalanced loads, single-phase and two-phase laterals, etc. Hence, the distribution systems need to be analyzed on the three-phase basis.

The paper reviews the three-phase load flow methods similar to the Gauss-Seidel formulation. These are viz. the implicit Z-bus method, modified Gauss-Seidel method, network topology based method, forward-backward substitution and ladder network theory. The algorithms for all these methods are discussed. The software programs are developed for the last three methods, which have been tested on a sample 8-bus system. The methods are compared with respect to different parameters. All the three tested methods are found to be quite accurate. The methods converged in three iterations. It is expected that, for a much larger and realistic network, the performance of these methods will be different.

The flexibility of all the five methods is compared. The ladder network theory and forward-backward substitution are more flexible for the changes in data as compared to the other methods. If any modification is to be made in the system data, then it is very easy in the ladder network theory and forward-backward substitution methods. But the same thing is difficult with the network topology based method, implicit Z-bus method and modified Gauss-Seidel method since the whole matrices need to be updated.

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